HEAT AND WATER TRANSFER FOR FRESHLY EXPOSED ROCK IN A VENTILATED CLOSED-END WORKING

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Heat and water transfer are considered for an unbounded isotropic rock body exposed to a ventilation jet of constant temperature. Formulas are derived that describe the distributions of the temperature and the water-transport potential in the rock. Equations are presented for the fluxes of heat and water from the rock into the air.

Calculations on the temperatures in closed-end mineworkings have been performed by reference to the heat transfer from the rock in a cavity of spherical form ventilated by air at constant or variable temperatures [1,2]. However, many studies have shown that mines also increase the humidity of the air because the rock dries out, and that the heat and mass transfer processes have their highest rates in freshly cut rock, when most of the water enters the air by evaporation from the walls.

In that case one can assume that the criterion for phase transformation in the rock is close to zero ($\varepsilon \approx 0$), and the differential equations for heat and mass transfer from the rock to the air will take the following form [3]:

$$\frac{\partial T(r,\ \tau)}{\partial \mathrm{Fo}_q} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\ \frac{\partial T}{\partial r}, \tag{1}$$

$$\frac{\partial V(r,\ \tau)}{\partial \mathrm{Fo}_m} = \frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\ \frac{\partial V}{\partial r} + \delta\ \frac{t_d - t_c}{\theta_d - \theta_e}\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\ \frac{\partial T}{\partial r}\right),\ 1 < r \tag{2}$$

with the boundary conditions

$$T(r,\ 0) = 0,\ \ V(r,\ 0) = 0, \tag{3}$$

for

$$r \to \infty:\ T(r,\ \tau) \to 0,\ V(r,\ \tau) \to 0, \tag{4}$$

$$\frac{\partial T(1,\ \tau)}{\partial r} + \mathrm{Bi}_q[1 - T(1,\ \tau)] - \frac{DR_0}{\lambda}[V(1,\ \tau) - 1] = 0, \tag{5}$$

$$\frac{\partial V(1,\ \tau)}{\partial r} + \delta\ \frac{t_d - t_c}{\theta_d - \theta_e}\ \frac{\partial T(1,\ \tau)}{\partial r} + \mathrm{Bi}_m[1 - V(1,\ \tau)] = 0, \tag{6}$$

where

$$T(r,\ \tau) = \frac{t(r,\ \tau) - t_d}{t_c - t_d};\ V(r,\ \tau) = \frac{\theta(r,\ \tau) - \theta_d}{\theta_e - \theta_d}; \tag{7}$$

$$D = \beta\rho(\theta_d - \theta_e)/(t_d - t_c);\ r = R/R_0;\ \mathrm{Fo}_q = a_q\tau/R_0^2;$$
$$\mathrm{Fo}_m = a_m\tau/R_0^2;\ \mathrm{Bi}_q = \alpha R_0/\lambda;\ \mathrm{Bi}_m = \beta R_0/\lambda_m. \tag{8}$$

The following are the reduced heat-transfer coefficient and Biot number

$$\tilde{\alpha} = \alpha_{re} = \alpha\left[1 + D\ \frac{\bar{V}(1,\ \tau)}{\alpha\bar{T}(1,\ \tau)}\right],\ \tilde{\mathrm{Bi}}_q = \tilde{\alpha}R_0/\lambda, \tag{9}$$

after which boundary condition (5) is converted to

$$\frac{\partial T(1,\ \tau)}{\partial r} - \tilde{\mathrm{Bi}}_q T(1,\ \tau) + \mathrm{Bi}_q + DR_0/\lambda = 0. \tag{10}$$

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We apply a Laplace–Carson transformation [4] to (1) and (2):

$$\bar{f}(r,\ p) = p \int\limits_0^{+\infty} \exp(-p\tau)\, f(r,\ \tau)\, d\tau.$$

We get the system

$$\frac{d^2\bar{T}}{dr^2} + \frac{2}{r}\ \frac{d\bar{T}}{dr} = \frac{R_0^2}{a_q}\ p\bar{T}, \tag{11}$$

$$\frac{d^2\bar{V}}{dr^2} + \frac{2}{r}\ \frac{d\bar{V}}{dr} + \delta\ \frac{t_{\text{d}} - t_{\text{c}}}{\theta_{\text{d}} - \theta_{\text{e}}} \left(\frac{d^2\bar{T}}{dr^2} + \frac{2}{r}\ \frac{d\bar{T}}{dr} \right) = \frac{R_0^2}{a_m}\ p\bar{V} \tag{12}$$

with the boundary condition for

$$r \to \infty:\ \bar{T}(r,\ p) \to 0,\ \bar{V}(r,\ p) \to 0, \tag{13}$$

$$\frac{d\bar{T}(1,\ p)}{dr} - \tilde{\text{Bi}}_q \bar{T}(1,\ p) + \text{Bi}_q + DR_0/\lambda = 0, \tag{14}$$

$$\frac{d\bar{V}(1,\ p)}{dr} + \delta\ \frac{t_{\text{d}} - t_{\text{c}}}{\theta_{\text{d}} - \theta_{\text{e}}}\ \frac{d\bar{T}(1,\ p)}{dr} + \text{Bi}_m [1 - \bar{V}(1,\ p)] = 0. \tag{15}$$

From (11) with (13) and (14) we get

$$\bar{T}(r,\ p) = \frac{(\text{Bi}_q + R_0 D/\lambda)\, b_q}{(1 + \tilde{\text{Bi}}_q)(b_q + \sqrt{p})\, r} \exp[-(r-1) R_0 \sqrt{p/a_q}], \tag{16}$$

where

$$b_q = \frac{\sqrt{a_q}}{R_0} (1 + \tilde{\text{Bi}}_q).$$

Substitution of (16) into (12) and (15) gives an equation whose solution is as follows for the conditions of (13) and (15):

$$\begin{aligned} \bar{V}(r,\ p) = {} & \frac{N b_m}{r(b_m + \sqrt{p})} \exp[-(r-1) R_0 \sqrt{p/a_m}] + \\ & + \frac{M b_q}{r(b_q + \sqrt{p})} \exp[-(r-1) R_0 \sqrt{p/a_m}] + \\ & + \frac{\text{E} b_q}{(\tilde{\text{Bi}}_q + 1)(a_q/a_m - 1)\, r(b_q + \sqrt{p})} \exp[-(r-1) R_0 \sqrt{p/a_q}], \end{aligned} \tag{17}$$

where

$$b_m = \frac{\sqrt{a_m}}{R_0} (1 + \text{Bi}_m); \tag{18}$$

$$\text{E} = \delta\ \frac{t_{\text{d}} - t_{\text{c}}}{\theta_{\text{d}} - \theta_{\text{e}}} (\text{Bi}_q + R_0 D/\lambda);$$

$$N = \frac{1}{\text{Bi}_m + 1} \left\{ \text{Bi}_m + \frac{\text{E}[\text{Bi}_m + a_q/a_m - (\text{Bi}_m + 1)\sqrt{a_q/a_m}]}{(a_q/a_m - 1)[(\text{Bi}_m + 1)\sqrt{a_m/a_q} - (\tilde{\text{Bi}}_q + 1)]} \right\}, \tag{19}$$

$$M = \frac{\text{E}(\tilde{\text{Bi}}_q a_q/a_m - \text{Bi}_m)}{(1 + \tilde{\text{Bi}}_q)(a_q/a_m - 1)[1 + \text{Bi}_m - (1 + \tilde{\text{Bi}}_q)\sqrt{a_q/a_m}]}. \tag{20}$$

Equations (16) and (17) are the transforms of the dimensionless temperature and water-transport potential in the rock. A standard formula is [5]

$$\frac{b \exp(-k\sqrt{p})}{b + \sqrt{p}} \risingdotseq \text{erfc}\, \frac{k}{2\sqrt{\tau}} - \exp(bk + b^2\tau)\ \text{erfc} \left(b\sqrt{\tau} + \frac{k}{2\sqrt{\tau}} \right). \tag{21}$$

We apply this to (16) and (17) to get an analytic expression for the dimensionless temperature and water-transport potential:

$$T(r,\ \tau) = \frac{t(r,\ \tau) - t_{\text{d}}}{t_{\text{c}} - t_{\text{d}}} = \frac{\text{Bi}_q + DR_0/\lambda}{(1 + \tilde{\text{Bi}}_q)\, r} \left\{ \text{erfc}\ \frac{r-1}{2\sqrt{\text{Fo}_q}} - \right.$$

$$- \exp[(r-1)(1+\tilde{\mathrm{Bi}}_q)+\mathrm{Fo}_q(1+\tilde{\mathrm{Bi}}_q)^2]\,\mathrm{erfc}\left[(1+\tilde{\mathrm{Bi}}_q)\sqrt{\mathrm{Fo}_q}+\frac{r-1}{2\sqrt{\mathrm{Fo}_q}}\right]\Bigg\}, \tag{22}$$

$$V(r,\tau)=\frac{\theta(r,\tau)-\theta_d}{\theta_e-\theta_d}=\frac{N}{r}\Bigg\{\mathrm{erfc}\,\frac{r-1}{2\sqrt{\mathrm{Fo}_m}}-\exp[(r-1)(\mathrm{Bi}_m+1)+$$
$$+\mathrm{Fo}_m(1+\mathrm{Bi}_m)^2]\,\mathrm{erfc}\left[(1+\mathrm{Bi}_m)\sqrt{\mathrm{Fo}_m}+\frac{r-1}{2\sqrt{\mathrm{Fo}_m}}\right]\Bigg\}+$$
$$+\frac{M}{r}\Bigg\{\mathrm{erfc}\,\frac{r-1}{2\sqrt{\mathrm{Fo}_m}}-\exp[(r-1)(\tilde{\mathrm{Bi}}_q+1)\sqrt{a_q/a_m}+\mathrm{Fo}_q(1+$$
$$+\tilde{\mathrm{Bi}}_q)^2]\,\mathrm{erfc}\left[(1+\tilde{\mathrm{Bi}}_q)\sqrt{\mathrm{Fo}_q}+\frac{r-1}{2\sqrt{\mathrm{Fo}_m}}\right]\Bigg\}+\frac{\mathrm{E}}{(1+\tilde{\mathrm{Bi}}_q)(a_q/a_m-1)r}\times$$
$$\times\Bigg\{\mathrm{erfc}\,\frac{r-1}{2\sqrt{\mathrm{Fo}_q}}-\exp[(r-1)(1+\tilde{\mathrm{Bi}}_q)+\mathrm{Fo}_q(1+\tilde{\mathrm{Bi}}_q)^2]\,\mathrm{erfc}\left[(1+\tilde{\mathrm{Bi}}_q)\sqrt{\mathrm{Fo}_q}+\frac{r-1}{2\sqrt{\mathrm{Fo}_q}}\right]\Bigg\}. \tag{23}$$

We further have the coefficients for the nonstationary heat and mass transfer [2]:

$$k_\tau=-\frac{\lambda}{R_0}\frac{\partial T(1,\tau)}{\partial r},\quad m_\tau=-\frac{\lambda_m}{R_0}\frac{\partial V(1,\tau)}{\partial r}. \tag{24}$$

Simple steps (6), (10), (22), and (23) with (24) give

$$k_\tau=\frac{\lambda \mathrm{Bi}_q+R_0D}{R_0}\left[1-\frac{\tilde{\mathrm{Bi}}_q}{1+\tilde{\mathrm{Bi}}_q}f(z_q)\right], \tag{25}$$

$$m_\tau=\frac{\lambda_m}{R_0}(\mathrm{Bi}_m-\mathrm{E})+\frac{\lambda_m}{R_0}\left\{\frac{\mathrm{E}\tilde{\mathrm{Bi}}_q}{1+\tilde{\mathrm{Bi}}_q}-\mathrm{Bi}_m\left[M+\frac{\mathrm{E}}{(\tilde{\mathrm{Bi}}_q+1)(a_q/a_m-1)}\right]\right\}f(z_q)-\frac{\lambda_m\mathrm{Bi}_m}{R_0}Nf(z_m), \tag{26}$$

where

$$z_q=(\tilde{\mathrm{Bi}}_q+1)\sqrt{\mathrm{Fo}_q},\quad z_m=(\mathrm{Bi}_m+1)\sqrt{\mathrm{Fo}_m},\quad f(z)=1-\exp(z^2)\,\mathrm{erfc}\,z. \tag{27}$$

Equations (25) and (26) give us the heat and water fluxes from

$$q=k_\tau(t_d-t_c),\quad m=m_\tau(\theta_d-\theta_e), \tag{28}$$

which are used in calculating the parameters of the air flow in the closed end of the workings. We show that one can obtain expressions for k_τ and m_τ for boundary conditions of the first kind, i.e., $t(R_0, \tau) = t_c$; for this purpose we pass to the limit in (10) and (25) and (26), where $\tilde{\mathrm{Bi}}_q \to \infty$ is $\mathrm{Bi}_q \to \infty$, which gives

$$k_\tau=\frac{\lambda}{R_0}\left(1+\frac{1}{\sqrt{\pi \mathrm{Fo}_q}}\right), \tag{29}$$

$$m_\tau=\frac{\lambda_m\mathrm{Bi}_m}{R_0}\left[1+\frac{\delta(t_d-t_c)}{(\theta_d-\theta_e)(1+\sqrt{a_q/a_m})}\right]-$$
$$-\frac{\lambda_m\delta(t_d-t_c)}{R_0(\theta_d-\theta_e)\sqrt{\pi\mathrm{Fo}_q}}+\frac{\lambda_m\mathrm{Bi}_m}{R_0(\mathrm{Bi}_m+1)}\left\{\frac{\delta(t_d-t_c)}{(\theta_d-\theta_e)(a_q/a_m-1)}\times\right.$$
$$\left.\times[\mathrm{Bi}_m+a_q/a_m-(\mathrm{Bi}_m+1)\sqrt{a_q/a_m}]-\mathrm{Bi}_m\right\}f(z_m), \tag{30}$$

where the formulas have been derived on the assumption that

$$\mathrm{Fo}_q\geqslant\left[\frac{\delta(t_d-t_c)}{\theta_d-\theta_e}\right]^2\frac{1}{\pi\mathrm{Bi}_m^2}. \tag{31}$$

These equations allow us to calculate the heat and mass fluxes from the rock into the air in the ventilation of a closed-end working with rapid evaporation from the walls.

NOTATION

R, spherical coordinate; R_0, radius of spherical end; $t(r, \tau)$, $\theta(r, \tau)$, temperature and water-transfer potential; t_d, t_c, temperature of the uncooled rocks and cooling jet, respectively; θ_d, θ_e, water-transfer potential and equilibrium value; c_q, c_T, specific heat and water capacity, respectively; a_q, thermal diffusivity; a_m, water transport coefficient; λ, λ_m, thermal conductivity and water conductivity, respectively; δ, thermogradient coefficient; erf x, probability integral.

LITERATURE CITED

1. O. O. Kremn'ov and E. M. Kozlov, "Coefficient of nonstationary heat transfer in a closed-end part of a mine working for a time-variable temperature in the ventilation flow," Dopov. Akad. Nauk Ukr. RSR, No. 3, 307 (1961).
2. A. N. Shcherban' and O. A. Kremnev, Scientific Principles for the Calculation and Control of Heat Conditions in Deep Pits, Volume 1 [in Russian], Izd. Akad. Nauk Ukr. SSR, Kiev (1959).
3. A. V. Lykov and Yu. A. Mikhailov, Theory of Energy and Matter Transport [in Russian], Izd. Akad. Nauk B SSR, Minsk (1959).
4. A. I. Lur'e, Operational Calculus [in Russian], GITTL, Moscow–Leningrad (1950).
5. H. Carslaw and J. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).

THERMOELASTICITY OF NONHOMOGENEOUS MEDIA

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A system of equations is derived for the coupled thermoelasticity of an anisotropic nonhomogeneous body, taking into account the generalized law of heat conduction, by the method of systems identification and with the aid of the Clausius–Duhem inequality.

In view of the extensive use of composite materials in various branches of technology, it becomes very important to study the properties of nonhomogeneous media.

The process of heat propagation through a nonhomogeneous medium will be simulated as follows: A system with the transfer function $G_{ij}(x_s)$, representing an anisotropic nonhomogeneous medium, receives a temperature gradient ∇t at the input and transmits a thermal flux $\vec{q}$ as its output signal. The output $\vec{q}$ of a linear process with the input ∇t is determined as the convolution integral

$$q_i = \int_0^{\tau} \nabla t(x_s, \tau - \tau_1) G_{ij}(x_s, \tau_1) d\tau_1 \quad (1)$$

or

$$q_i = \int_0^{\tau} \nabla t(x_s, \tau_1) G_{ij}(x_s, \tau - \tau_1) d\tau_1. \quad (2)$$

We introduce the notation $Z = q_i$ and $Y = \nabla t$. We then define the correlation function φ_{zy} which describes the coupling between quantities Z and Y

$$\varphi_{zy} = \lim_{\tau_3 \to \infty} \frac{1}{2\tau_3} \int_{-\tau_3}^{\tau_3} Z(x_s, \tau) Y(x_s, \tau - \tau_1) d\tau. \quad (3)$$

We analogously define the autocorrelation function φ_{yy} as the average product of the value of signal $Y(x_s, \tau)$ and its value at time $(\tau - \tau_2)$

$$\varphi_{yy} = \lim_{\tau_3 \to \infty} \frac{1}{2\tau_3} \int_{-\tau_3}^{\tau_3} Y(x_s, \tau) Y(x_s, \tau - \tau_2) d\tau. \quad (4)$$

We will consider an input function Y of the "white noise" kind, whose autocorrelation function is a delta function. A preliminary transformation of function (3) with relation (1) taken into account yields

$$\varphi_{zy}(x_s, \tau_2) = G_{ij}(x_s, \tau_2). \quad (5)$$

With the aid of relation (5), relation (2) transforms to

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